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Development and Research on Innovative Vocational Education Skills

SENSOR FUSION EXPERT

SFE.U4.E2 - MEASUREMENT MODELS

Data and Sensor Fusion Applications, Use Cases and Real-Life Examples

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LEARNING OBJECTIVES

The student is able to …

SFE.U4.E2.PC1	The student understands what measurement models are.
SFE.U4.E2.PC2	The student understands the logic behind each measurement model.
SFE.U4.E2.PC3	The student knows the different use cases of each type of measurement model.
SFE.U4.E2.PC4	The student is able to select an adequate measurement model to a specific situation or problem.
SFE.U4.E2.PC5	The student can employ and evaluate a measurement model.

Introduction

- A model is a mathematical formula that creates a relationship with the amount of interest and measurements in a systematic way.
- Sensors suffer from several limitations that we need to consider in models;
- It is important to take into account uncertainties in measurements such as sensor noise or measurement noise.
- The problem is that measurement noise is difficult to quantify analytically.
- So a suitable approach is to use statistical modeling.

Introduction

- In short, the real noise value cannot be observed or measured, but its statistical properties can.
- With this, sensor fusion methods can be designed
- These methods are able to minimize (statistically) the effect of noise.

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Introduction

- These models are composed of:
    - Basic model;
    - Vector model,
    - Multiple measurements and measurement stacking
    - Gaussian measurement noise.

Introduction

- It's a very simplistic model
- It's quite generic
- The objective of this model is to relate the measurement of yn with the unknown x so that the uncertainty is taken into account

The Model

$$y_n = g_n(\mathbf{x}) + r_n.$$

- The quantity measured in yn is found on the left side of the equation.
- On the right side there are two terms
  - A function that will relate the quantities of interest x to the measurement yn and a noise term rn.

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The Model

- However, as shown above, measurement noise can be considered a random variable.
- Thus, rn will have to follow some kind of probability density function (pdf), that is, it will have to:

$$r_n \sim p(r_n),$$

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The Model

- Which reads as:
  - "rn is distributed according to p(rn)",
    - p(rn) denotes the corresponding pdf.
- At this point, we will not be concerned with any particular form of p(rn), but we will just assume that the rns are zero-mean independent random variables with variance σ 2 r, n.

The Model

- So it will be necessary to do this:

$$\mathrm{E}\{r_n\} = 0,$$
$$\mathrm{var}\{r_n\} = \mathrm{E}\{r_n^2\} - (\mathrm{E}\{r_n\})^2 = \sigma_{r,n}^2,$$
$$\mathrm{Cov}\{r_m, r_n\} = \mathrm{E}\{r_m r_n\} - \mathrm{E}\{r_m\}\,\mathrm{E}\{r_n\} = 0, \quad (m \neq n),$$

The Model

- Where E {rn} and var {rn} denote the expected value and the variance of rn, respectively.
- Note that measurement noise variance may vary for different measurements.
  - As indicated by the subscript n in σ 2 r, n
    - (eg if different measurements are taken by different sensors).

Introduction

- The basic model presented above only considers scalar measures.
- Many sensors can provide measurements with vector values
  - An example of this are accelarometers
  - Accelarometers provide full 3D acceleration measurements at each sampling instant.
- Therefore, an extension of a basic model can be created to take into account the measured values in vector.

The Model

- In this case, the vector model can be described as follows:

$$\mathbf{y}_n = \mathbf{g}_n(\mathbf{x}) + \mathbf{r}_n,$$

- yn is an dy-dimensional column vector.
- In this case, the measurement noise rn becomes a multivariate random variable with pdf.

$$\mathbf{r}_n \sim p(\mathbf{r}_n).$$

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The Model

- As for the scalar case, p(rn) can have any arbitrary form.
- For simplicity, it can again be assumed that the rns are zero-mean independent random variables with a covariance matrix Rn, that is,

$$\mathrm{E}\{\mathbf{r}_n\} = 0,$$
$$\mathrm{Cov}\{\mathbf{r}_n\} = \mathrm{E}\{\mathbf{r}_n\mathbf{r}_n^{\mathsf{T}}\} - \mathrm{E}\{\mathbf{r}_n\}\,\mathrm{E}\{\mathbf{r}_n\}^{\mathsf{T}} = \mathbf{R}_n,$$
$$\mathrm{Cov}\{\mathbf{r}_m, \mathbf{r}_n\} = \mathrm{E}\{\mathbf{r}_m\mathbf{r}_n^{\mathsf{T}}\} - \mathrm{E}\{\mathbf{r}_m\}\,\mathrm{E}\{\mathbf{r}_n\}^{\mathsf{T}} = 0, \quad (m \neq n).$$

Conclusion

- As can be seen, the scalar model is just a special case of the vector model, where measurement and noise are scalar (ie, dy = 1).
- Thus, it is generally more common to work with the vector model, but sometimes it is more instructive to work with the scalar model.

Introduction

- To perform a sensor fusion with multiple measurements, multiple measurements are required;
- These can be:
  - From a sensor at different points in time (repeated measurements),
  - From different sensors;
  - Or both.

Introduction

- If we get a total of N measures y1, y2,. . . , yN, let's refer to the set of all measurements using the notation:
  - y1: N = {y1, y2,. . . , yN} for the scalar case
  - y1: N = {y1, y2,. . . , yN} for the vector case

The Model

- Sometimes it is also useful to write the complete set of measurements together with the measurement model more compactly.
- This can be achieved by stacking the measurements into a single measurement vector, which produces a much more compact model of the formula.

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) + \mathbf{r}.$$

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The Model

- For scalar measurements of the formula below,

$$y_n = g_n(\mathbf{x}) + r_n.$$

- We have to:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}, \ \mathbf{g}(\mathbf{x}) = \begin{bmatrix} g_1(\mathbf{x}) \\ g_2(\mathbf{x}) \\ \vdots \\ g_N(\mathbf{x}) \end{bmatrix}, \text{ and } \mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix}.$$

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The Model

- Furthermore, the overall noise covariance R for this model is a diagonal matrix of the formula:

$$\mathbf{R} = \mathrm{Cov}\{\mathbf{r}\} = \begin{bmatrix} \sigma^2_{r,1} & 0 & \dots & 0 \\ 0 & \sigma^2_{r,2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & \sigma^2_{r,N} \end{bmatrix}.$$

The Model

- For vector measurements, y, g (x) and r are constructed in the same way as for the scalar case, ie:

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{bmatrix}, \quad \mathbf{g}(\theta) = \begin{bmatrix} g_1(\mathbf{x}) \\ g_2(\mathbf{x}) \\ \vdots \\ g_N(\mathbf{x}) \end{bmatrix}, \quad \text{and } \mathbf{r} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_N \end{bmatrix}.$$

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The Model

- The measurement noise covariance is a diagonal block matrix with the individual covariance matrices Rn on the diagonal, ie,

$$\mathbf{R} = \mathrm{Cov}\{\mathbf{r}\} = \begin{bmatrix} \mathbf{R}_1 & 0 & \dots & 0 \\ 0 & \mathbf{R}_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & \mathbf{R}_N \end{bmatrix}.$$

The Model

- A common assumption is that the measurement noise is zero-mean Gaussian noise.
- In this case, the noise pdf is the Gaussian (multivariate) distribution given by:

$$p(\mathbf{r}) = \frac{1}{(2\pi)^{M/2}|\mathbf{R}|^{1/2}} \exp\left(-\frac{1}{2}\mathbf{r}^{\top}\mathbf{R}^{-1}\mathbf{r}\right),$$

The Model

- Generally, N (z; μ, Σ) denotes the multivariate pdf of an M-dimensional Gaussian random variable z with mean μ and covariance matrix Σ defined as:

$$\mathcal{N}(\mathbf{z};\boldsymbol{\mu},\boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{M/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{z}-\boldsymbol{\mu})^{\mathsf{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{z}-\boldsymbol{\mu})\right).$$

- Assuming that the measurement noise is Gaussian is indeed reasonable in many (but not all) applications.

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The aim of the Blueprint is **to support an overall sectoral strategy and to develop concrete actions to address short and medium term skills needs.**

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